

Closing Tues: HW 14.2 (part 2)  
 Closing Thur: HW 14.3/4 (last HW)  
 Final: Sat, March 10, 5:00-7:50pm,  
 Final Room is based on quiz section  
 For BC/BD, AC/AD: PAA A102  
 For AA/AB, BB: PAA A118  
 For BA: PAA A110

## 14.4 More Applications

### 1. Cost Breakdown (HW 14.3/1-2)

Suppose the cost to produce ONE item is given by:

$$C(x, y) = 3x^2 + 4y^2 + 5xy + 10,$$

where

$x$  = cost for 1 hour of labor, and  
 $y$  = cost for 1 pound of materials.

### Entry Task:

The current hourly rate for labor is \$20 and material is \$55 per pound.

How will a \$1 per hour raise for labor affect the cost to produce 1 item?

$$C_x = 6x + 5y \quad \frac{\text{dollars}}{\text{labor hour}}$$

$$C_y = 8y + 5x \quad \frac{\text{dollars}}{\text{pound}}$$

$$C_x(20, 55) = 6(20) + 5(55)$$

$$= 395 \quad \frac{\$}{\text{labor hour}}$$

IF LABOR COSTS GO UP \$1/hr  
 THEN IT WILL COST \$395 MORE  
 TO PRODUCE ONE ITEM.

## 2. Marginal Productivity (14.3/5-6)

Suppose that the number of crates of a particular fruit produced is

$$z = \frac{9xy - 0.0002x^2 - 5y}{0.03x + 4y} \leftarrow N \leftarrow D$$

where

$x$  = number of hours of labor, and

$y$  = number of acres of the crop.

$$N = 9xy - 0.0002x^2 - 5y$$

$$N_x = 9y - 0.0004x$$

$$D = 0.03x + 4y$$

$$D_x = 0.03$$

$$\frac{\partial z}{\partial x} = \frac{DN' - ND'}{D^2}$$

$$= \frac{(0.03x + 4y)(9y - 0.0004x) - (9xy - 0.0002x^2 - 5y)(0.03)}{(0.03x + 4y)^2}$$

Find the marginal productivity for hours of labor when  $x = 100$  and  $y = 200$ . Interpret your answer

$$\text{AT } x = 100, y = 200$$

$$\frac{\partial z}{\partial x} = \frac{(803)(1799.96) - (178998)(0.03)}{(803)^2}$$

$$\frac{\partial z}{\partial x} = \frac{\text{Crates produced}}{\text{labor hours}} = \text{MARGINAL PRODUCTIVITY} \approx 2.23322 \frac{\text{crates}}{\text{labor hour}}$$

IF LABOR HOURS IS INCREASED BY ONE THEN ABOUT 2.23 more crates of fruit will be produced.

### 3. Revenue/Cost

Assume you manufacture and sell two products, A and B.

Let

$x$  = thousands of units of A, and  
 $y$  = thousands of units of B.

You know from past years that your cost (in thousand dollars) is given by

$$C(x, y) = 2x^2 - 2xy + y^2 - 9x - 10y + 11$$

And you know:

- Product A sells for \$5.00/item, and
- Product B sells for \$8.00/item.

What is the maximum profit?

$$R(x, y) = 5x + 8y \quad \leftarrow \text{REVENUE!}$$

PROFIT IS MAXIMIZED WHEN WE PRODUCE AND SELL  
16 thousand of product A, 25 thousand of product B, and  
MAX PROFIT IS

326 thousand dollars

$$P(x, y) = \text{PROFIT}$$

$$= R(x, y) - C(x, y)$$

$$= [5x + 8y] - [2x^2 - 2xy + y^2 - 9x - 10y + 11]$$

$$= 5x + 8y - 2x^2 + 2xy - y^2 + 9x + 10y - 11$$

$$P(x, y) = -2x^2 + 2xy - y^2 + 14x + 18y - 11$$

$$P_x = -4x + 2y + 14 \stackrel{?}{=} 0 \Rightarrow 2y = 4x - 14$$

$$P_y = 2x - 2y + 18 \stackrel{?}{=} 0$$

$$y = 2x - 7$$

$$2x - 2(2x - 7) + 18 \stackrel{?}{=} 0$$

$$2x - 4x + 14 + 18 \stackrel{?}{=} 0$$

$$32 = 2x$$

$$x = 16$$

$$y = 2(16) - 7 = 25$$

$$P(16, 25) = -2(16)^2 + 2(16)(25) - (25)^2 + 14(16) + 18(25) - 11 = 326$$